CORPUS CHRISTI COLLEGE SEQUERE DOMINUM	Corpus Christi College
	Year 12 Mathematics Methods
	2019 Test 5

Name: MARKING KEY.

Date: 12 Sept 2019
Time: 45 minutes
Total: **18** + 27 = 45 marks
Weight: 8%

TOPICS: Continuous Random Variables, Normal Distribution, Sampling, Sample Proportions

SECTION A – NON CALCULATOR

INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Calculators are not allowed
- No Notes allowed

You may assume the following z scores for normal distributions and confidence intervals

- **For 68% of scores** $-1 \leq z \leq 1$
- **For 95% of scores** $-2 \leq z \leq 2$
- **For 99.7% of scores** $-3 \leq z \leq 3$

Student Reflection

Q1	Q2,5	Q3,7	Q4	Q6	Total
Uniform Distribution	Normal Distribution	Confidence Intervals	Sample proportions	Continuous Random Variables	
4	12	16	5	8	**45**

What went well:
I did well at…

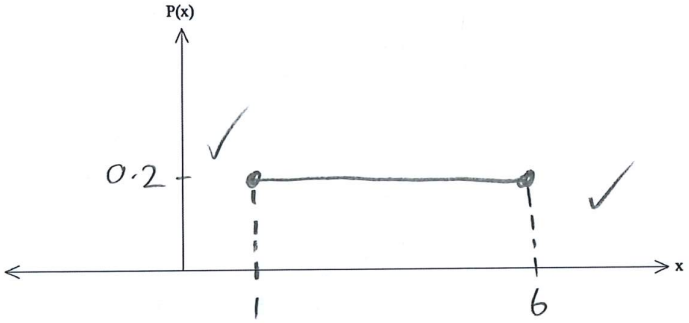
Areas for development:
I need to improve…

1. [4 marks]

Anna arrives at 8.38 am, two minutes early for her maths methods class and knows that it is equally likely for her maths teacher to arrive at class anywhere from 1 minute to 6 minutes later.

Let the continuous random variable X be the number of minutes taken for Anna's teacher to arrive after 8.38 am.

a) Draw a graph on the axes below that shows the probability density function of the random variable X. [2]



b) What is the probability that Anna's maths teacher arrives after 8.40 am? [1]

$\frac{4}{5}$ ✓

c) What is the probability that Anna's maths teacher arrives before 8.42 am given that he arrives after 8.40 am? [1]

$\frac{P(2<x<4)}{P(x>2)} = \frac{1}{2}$ ✓

4

2. [7 marks]

The maximum temperatures of Perth days in the month of April can be modelled using a normal distribution with a mean of 26 °C and a standard deviation of 3.

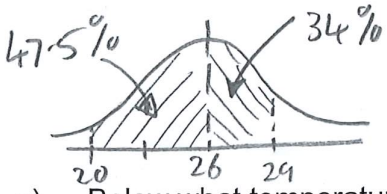
Using this model, answer the following.

a) If the first day of April had a standardized score of -1.25, what was the maximum temperature on this day? [1]

$26 - 3 \times 1.25$

$26 - 3.75 = 22.25\,^{\circ}C$ ✓

b) What is the probability that the maximum temperature of an April day will be between 20 °C and 29 °C? [1]



$47.5 + 34 = 81.5\,\%$ ✓

c) Below what temperature do the lowest 16% of the daily maximums in April lie? [1]

$23\,^{\circ}C$

d) How many April days in the next decade would you expect to have maximum temperatures above 32 °C?
(Note: there are 30 days in the month of April) [2]

300×0.025 ✓ $= 300 \div 10 \div 2 \div 2 = 7.5$ ✓

7 or 8 days

e) The lowest recorded maximum for an April day is 16.3°C. Is this consistent with the use of the described model? Explain your answer. [2]

✓ We would expect 0.15% of the days to be below 17°C.

✓ Hence a lowest temperature of 16.3°C is consistent with the model.

3. [7 marks]

a) A random sample of size n_1 was taken and the proportion of people who had cycled in the last week was m.

Determine a 68% confidence interval for the proportion of the population who had cycled in the last week in terms of n_1 and m. [2]

$$m - \sqrt{\frac{m(1-m)}{n_1}} \leq p \leq m + \sqrt{\frac{m(1-m)}{n_1}}$$ ✓✓

b) A new sample of size n_2 was taken and the proportion of people who had cycled in the last week was again m. When a 95% confidence interval was determined it was found to be the same as the interval determined in part (a).

(i) Is n_2 larger or smaller than n_1? Explain [2]

n_2 is larger, to compensate for the larger % value.

✓ ✓

(ii) What is the relationship between n_1 and n_2? [3]

$$\sqrt{\frac{m(1-m)}{n_1}} = 2\sqrt{\frac{m(1-m)}{n_2}}$$ ✓

$$\frac{m(1-m)}{n_1} = 4\frac{m(1-m)}{n_2}$$ ✓

$$\frac{n_2}{n_1} = 4$$ ✓

n_2 is 4 times larger than n_1

7

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SECTION B – CALCULATOR ALLOWED

INSTRUCTIONS:

- Show all necessary working out
- Approved Formula sheet allowed
- Scientific and CAS Calculators are allowed
- One A4 page of notes (both sides) is allowed

4. [5 marks]

It is known that 12% of the population are left handed.

a) Describe the distribution of the proportions of left handers in samples of size 500. [2]

Approximately normal with ✓

mean of sample proportions = 0.12

σ of sample proportions = $\sqrt{\frac{0.12(0.88)}{500}} = 0.0145$ ✓

b) 500 major league baseballers were surveyed and it was found that 95 of them were left handed. Comment on this result. [3]

$\hat{p} = \frac{95}{500} = 0.19$ ✓

$z = \frac{0.19 - 0.12}{0.0145} = 4.817$ ✓

Sample proportion is almost $5 \times \sigma$ above population proportion.

It appears that there is a greater proportion of left handers in MLB compared to the proportion of left handers in the population. ✓

(or similar)

5. [5 marks]

The horn length of adult black rhinos is normally distributed with 38% of adult black rhinos having a horn length above 75 cm and 12% of adult black rhinos having a horn length below 61 cm.

Above what length are the longest 10% of adult black rhino horns?

$\text{invnormal}(R, 0.38, 1, 0) \Rightarrow z = 0.3055$ ✓

$\text{invnorm}(L, 0.12, 1, 0) \Rightarrow z = -1.17499$ ✓

$$\left.\begin{aligned} 0.3055 &= \frac{75-\mu}{\sigma} \\ -1.175 &= \frac{61-\mu}{\sigma} \end{aligned}\right\} \Rightarrow \begin{aligned} \mu &= 72.11 \quad ✓ \\ \sigma &= 9.46 \quad ✓ \end{aligned}$$

$\text{invnorm}(R, 0.1, 9.46, 72.11) \Rightarrow K = 84.2$.

84.2 cm ✓

6. [8 marks]

The time X minutes for a meal to be delivered by an uber eats driver is modelled using a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(x-30)^2 & : \ 0 < x < 30, \\ 0 & : \ \text{elswhere} \end{cases}$$

a) Find the value of k [2]

$$\int_0^{30} k(x-30)^2 \, dx = 1 \quad \checkmark$$

$$9000k = 1$$

$$k = \frac{1}{9000} \quad \checkmark$$

b) What is the probability of the driver delivering a meal within 15 minutes? [2]

$$\int_0^{15} \frac{1}{9000}(x-30)^2 \, dx = \frac{7}{8} \quad \checkmark \quad \checkmark$$

c) Calculate the mean delivery time for the driver [2]

$$\text{Mean} = \int_0^{30} x \cdot \frac{1}{9000}(x-30)^2 \, dx = 7.5 \quad \checkmark \quad \checkmark$$

d) Calculate the standard deviation of the delivery time for the driver. [2]

$$\sigma^2 = \int_0^{30} \frac{1}{9000}(x-30)^2 (x-7.5)^2 \, dx \quad \checkmark$$

$$\sigma^2 = 33.75 \Rightarrow \sigma = 5.809 \quad \checkmark \quad \left(\text{or } \sigma = \frac{3\sqrt{15}}{2}\right)$$

7. [9 marks]

In a random sample of 200 Year 12 ATAR students, it was found that 28 of the students received extra tutoring outside of school.

a) Calculate the sample proportion of these students who received extra tutoring outside of school. [1]

$\frac{28}{200} = 0.14$ ✓

b) Calculate the 90% confidence interval for the population proportion and interpret your answer. [3]

$0.14 - 1.645\sqrt{\frac{0.14 \times 0.86}{200}} \leq p \leq 0.14 + \sqrt{\frac{0.14 \times 0.86}{200}}$

$0.0996 \leq p \leq 0.1804$ ✓✓

90% certain that this confidence interval contains the population proportion.

c) A second survey of Year 12 ATAR students is planned; however, it is decided that the 90% confidence interval should involve a maximum margin of error of 3%. Determine the sample size required for such a survey. [3]

$1.645\sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.03$ ✓

solve on CAS $\Rightarrow n \geq 751.54$ ✓

$n = 752$ ✓

Accept $1.645\sqrt{\frac{0.14 \times 0.86}{n}} \leq 0.03$

$n \geq 361.94 \Rightarrow n = 362.$

Also Accept $n \geq 362.006 \Rightarrow n = 363$

(d) If ten surveys were taken and for each a 90% confidence interval for the population proportion was calculated, determine the probability that at most seven of the intervals included the true value of the population proportion. [2]

$X \sim Bin(10, 0.9)$ ✓

$P(X \leq 7) = 0.0702.$ ✓